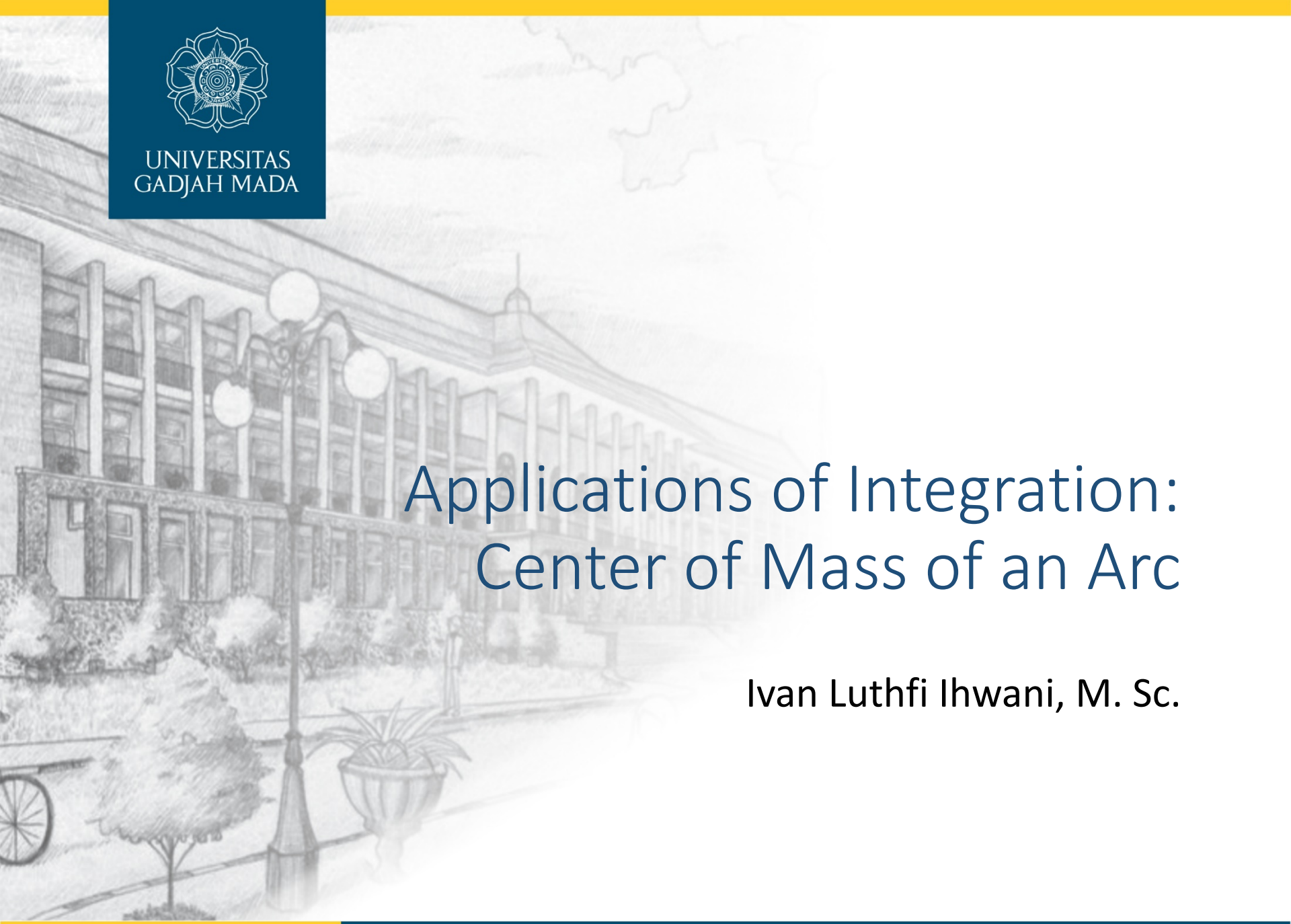

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Applications of Integration: Center of Mass of an Arc

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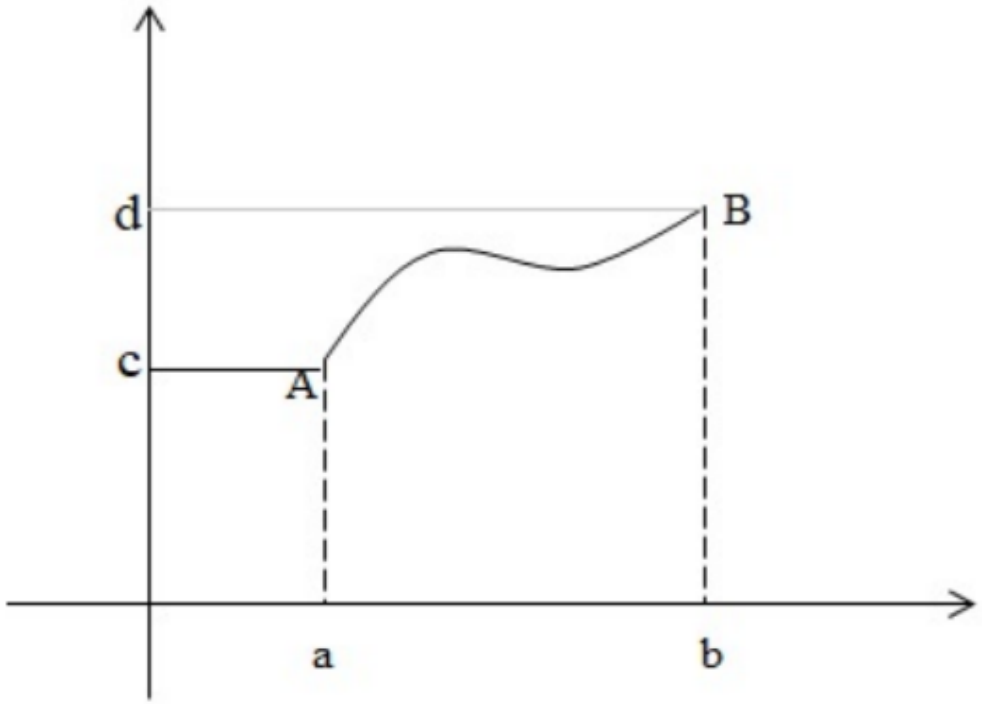
Center of Mass of an Arc

- Given a continuous and differentiable arc, K, with curve $y = f(x)$ and a constant of mass density ρ. Endpoints of arc K are $A(a, c)$ and $B(b, d)$.



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Center of Mass of an Arc

- If we cut the arc into small pieces, then we obtain the *i*-th piece of the arc with the mass $m_i = \rho\, ds_i$, where ds_i is the derivative of the *i*-th arc (read Slide about "Length of the Arc").
- Next, by the definition of center of mass and definite integral, the coordinate of the center of mass of K is $(\bar{x}, \bar{y})$.

$$\bar{x} = \frac{\int x\rho\ ds}{\int \rho\ ds} = \frac{\int x\ ds}{\int ds}$$

and

$$\bar{y} = \frac{\int y\rho\ ds}{\int \rho\ ds} = \frac{\int y\ ds}{\int ds}$$

Where ds is the derivative of the arc. (formula for ds can be found in the lecture slide about "Length of the Arc")

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Example

Find the center of mass of an arc where the curve is $\begin{cases} x = 5\cos t \\ y = 5\sin t \end{cases}$ and t is start from 0 to $\frac{\pi}{2}$.

Solution:

By the formula, ds is

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \sqrt{25\sin^2 t + 25\cos^2 t}\; dt = 5\, dt$$

Suppose that the center of mass of this arc is $(\bar{x}, \bar{y})$.

Example

Solution (continuation):

Since the arc is symmetric around the straight line $y = x$ and homogeneous, then the center of mass of this arc is located on line $y = x$. So the center of mass will be $(\bar{x}, \bar{x})$ where

$$\bar{x} = \frac{\int_0^{\frac{\pi}{2}} 5\cos t \cdot 5 \, dt}{\int_0^{\frac{\pi}{2}} 5 \, dt} = \frac{25[\sin t]_0^{\frac{\pi}{2}}}{5[t]_0^{\frac{\pi}{2}}} = \frac{5}{\pi}$$

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Exercises

1) Determine the center of mass of an arc $r = 6\sin\theta + 4\cos\theta$, where θ is start from 0 to $\frac{\pi}{2}$.
2) Determine the center of mass of a cardiode arc $r = 1 + \cos\theta$, where θ is start from 0 to $\frac{\pi}{2}$.

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Thank You