Applications of Integration: Center of Mass of Surface Revolution

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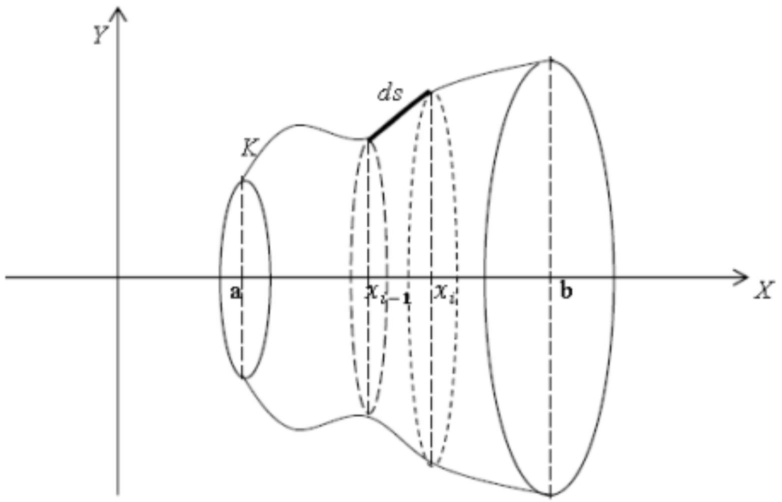
Center of Mass of Surface Revolution

- Given an arc, K, with a continuous and differentiable curve $y = f(x)$ on the interval $[a, b]$. If we revolve K around the x-axis, we will have the surface of revolution.



Center of Mass of Surface Revolution

- Since the arc is revolved around x, the center of mass and the mass of surface revolution consecutively are $(\bar{x}, 0)$ and $m = \rho(2\pi y ds)$. Therefore, the coordinate of the mass point will be $(\bar{x}, 0)$ where

$$\bar{x} = \frac{\int_a^b xy\ ds}{\int_a^b y\ ds}$$

- Similarly, if the arc, K, with a continuous and differentiable curve $x = g(y)$ on the interval $[c, d]$ is revolved around the y-axis, then the coordinate of the mass point will be $(0, \bar{y})$ where

$$\bar{y} = \frac{\int_c^d xy\ ds}{\int_c^d x\ ds}$$

Example

Find the center of mass of surface revolution which come from an arc of circle $y = \sqrt{1-x^2}$ revolved around the x-axis.

Solution:

Here we have

$$\frac{dy}{dx} = \frac{-2x}{2\sqrt{1-x^2}} \Longrightarrow ds = \frac{1}{\sqrt{1-x^2}}dx$$

The center of mass of the surface revolution will be $(\bar{x}, 0)$ where

$$\bar{x} = \frac{\int_{-1}^{1} x\sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}\, dx}{\int_{-1}^{1} \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}\, dx} = \frac{\left[\frac{1}{2}x^2\right]_{-1}^{1}}{[x]_{-1}^{1}} = \frac{\frac{1}{2} - \frac{1}{2}}{1+1} = \frac{0}{2} = 0$$

(The formula for ds can be found on the Slide about "the Arc Length")

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Exercise

Find the center of mass of surface revolution if the arc:

1) $y = x^2$ from $(-1,1)$ to $(-2, 4)$ is revolved around the y-axis.

2) $y = \frac{1}{2}x^2$ from $x = 0$ to $x = 1$, revolved around y-axis.

Thank You