

Numerical Analysis II

Homework 1

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1. If $|a_{ii} - \lambda| > \sum_{j=1, j \neq i}^n |a_{ij}|$, then the matrix $A - \lambda I$ is diagonally dominant. Using this idea, prove Gershgorin's Theorem.
 2. (a) Sketch the Gershgorin disks for the matrix

$$A = \begin{bmatrix} 0 & 2 & -1 \\ -2 & -10 & 0 \\ -1 & -1 & 4 \end{bmatrix}$$

and give a bound for the spectral radius, $\rho(A)$.

- (b) Determine an upper and lower bound for $\rho(A)$ using $\|A\|_1$, where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

and then repeat using Gershgorin's Theorem.

3. Use Gershgorin's Theorem to prove that a diagonally dominant matrix does not have 0 as an eigenvalue and is therefore nonsingular.
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