Numerical Analysis I
Homework 3

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1. Use of the familiar quadratic formula

$$x = \frac{1}{2a}\left(-b \pm \sqrt{b^2 - 4ac}\right)$$

will cause a problem when the quadratic equation $x^2 - 10^5 x + 1 = 0$ is solved with a machine that carries only eight decimal digits. Investigate the example, observe the difficulty, and propose a remedy.

2. Given an n-by-n nonsingular matrix A, how do you efficiently solve the problem $A^k\mathbf{x} = b$, where k is a positive integer, using the decomposition $A = LU$, which is assumed to be available. You should
   (a) Describe your algorithm,
   (b) Present your algorithm in pseudocode, and
   (c) Give the required flops. Note that one flop = one addition + one multiplication.

3. Let

$$f(x) = \begin{cases} (x-\alpha)^{2/3}, & x \geq \alpha \\ (\alpha - x)^{2/3}. & x \leq \alpha \end{cases}$$

   (a) Write down Newton's method for this function.
   (b) Will Newton's method converges?
   (c) If so, what is the order of convergence (or rate if the order is 1)?

4. Use the following example

$$f(x) = \sqrt{x^2 + 1} - 1,$$

whose value needs to be evaluated for x near zero, to explain what the loss of significance means and propose a way to avoid it in subtraction.

5. Consider the following system of nonlinear equations

$$\begin{cases} x + y + z = 3, \\ x^2 + y^2 + z^2 = 5, \\ e^x + xy - yz = 1. \end{cases} \tag{1}$$

Give a complete algorithm of Newton's method for the nonlinear systems given by (1) using an initial vector $\mathbf{x}^{(0)} = \left(x^{(0)}, y^{(0)}, z^{(0)}\right)^T$. The algorithm should include some input and output data, and stopping conditions, etc. Also, write explicitly down the Jacobian matrices J.

6. Find the lower triangular matrix L and an upper triangular matrix U so that

$$LU = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix},$$

and find the determinant of LU.
