Numerical Analysis II
Homework 4

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1. Derive the following two formulas for approximating derivatives and show that they are both $O\left(h^4\right)$ by establishing their error terms.

$$f'(x) \approx \frac{1}{12h}\left[-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)\right] \tag{1}$$

$$f''(x) \approx \frac{1}{12h^2}\left[-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)\right] \tag{2}$$

2. Derive the following two formulas for approximating the third derivative. Find their error terms. Which formula is more accurate?

$$f'''(x) \approx \frac{1}{h^3}\left[f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)\right] \tag{3}$$

$$f'''(x) \approx \frac{1}{2h^3}\left[f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)\right] \tag{4}$$

3. Show how to use Richardson extrapolation if

$$L = \varphi(h) + a_1 h + a_3 h^3 + a_5 h^5 + \ldots$$

4. Derive a numerical differentiation formula of order $O\left(h^4\right)$ by applying Richardson extrapolation to

$$f'(x) = \frac{1}{2h}\left[f(x+h) - f(x-h)\right] - \frac{h^2}{6}f'''(x) - \frac{h^4}{120}f^{(5)}(x) - \ldots$$

   Give the error term of order $O\left(h^4\right)$.
