Numerical Analysis I
Homework 5

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1. Consider the matrices below

$$A = \begin{bmatrix} 2.0 & 5.0 & -4.0 \\ 3.0 & 10.0 & -7.0 \\ -3.0 & 6.0 & 1.0 \end{bmatrix}, \quad \text{and} \quad A^{-1} = \begin{bmatrix} 26.0 & -14.5 & 2.5 \\ 9.0 & -5.0 & 1.0 \\ 24.0 & 13.5 & 2.5 \end{bmatrix}.$$

   (a) What is $\|A\|_1$?

   (b) What is the condition number of A in that norm?

   (c) Suppose we want to solve $A\mathbf{x} = \mathbf{b}$, where the values in $\mathbf{b}$ are measurements such at the relative error in $\mathbf{b}$ is 0.00001. How large the relative error in $\mathbf{x}$ due to measurement error be?

2. Prove that if A is unit row diagonally dominant, i.e., $a_{ii} = 1 > \sum_{j=1, j\neq i} |a_{ij}|$, $(1 \leq i \leq n)$ then the Richardson iteration converges for any initial guess.

3. Consider the following iterative method

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \gamma \left(\mathbf{b} - A\mathbf{x}^{(k)}\right), \tag{1}$$

   where $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, and $\gamma = \frac{2}{\rho(A)}$, where $\rho(A)$ is the spectral radius of A. Will this algorithm converge to the solution of $A\mathbf{x} = \mathbf{b}$ for any initial guess $\mathbf{x}^{(0)}$? Justify your answer using some theory we learned in class. Employing the method numerically is NOT an acceptable answer. Note that your answer is not dependent of $\mathbf{b}$ and therefore its explicit form is omitted.
