Numerical Analysis II
Homework 5

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1. Verify that the function $x(t) = \frac{t^2}{4}$ solves the initial-value problem

$$\begin{cases} x' = \sqrt{x}, \\ x(0) = 0. \end{cases} \tag{1}$$

   Apply the Taylor-Series method of order 1, and explain why the numerical solution differs from the solution $\frac{t^4}{4}$.

2. Derive the modified Euler's method

$$x(t+h) = x(t) + hf\left(t + \frac{1}{2}h, x(t) + \frac{1}{2}hf(t, x(t))\right)$$

   by performing Richardson's extrapolation on Euler's method using step size h and $\frac{h}{2}$.
   Hint: assume the error term is Ch^2.

3. Prove that when the fourth-order Runge-Kutta method is applied to the problem $x' = \lambda x$, the formula for advancing this solution will be

$$x(t+h) = \left[1 + h\lambda + \frac{1}{2}h^2\lambda^2 + \frac{1}{6}h^3\lambda^3 + \frac{1}{24}h^4\lambda^4\right] x(t).$$
