Numerical Analysis II
Homework 7

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1. Discuss these multistep methods in light of Theorem 1 (p. 558), on multistep method stability and consistency.
   (a) $x_n - x_{n-2} = 2hf_{n-1}$
   (b) $x_n - x_{n-2} = h\left[\frac{7}{3}f_{n-1} - \frac{2}{3}f_{n-2} + \frac{1}{3}f_{n-3}\right]$
   (c) $x_n - x_{n-1} = h\left[\frac{3}{8}f_n + \frac{19}{24}f_{n-1} - \frac{5}{24}f_{n-2} + \frac{1}{24}f_{n-3}\right]$
2. Determine the numerical characteristic of the multistep method whose equation is
$$x_n + 4x_{n-1} - 5x_{n-2} = h\left[4f_{n-1} + 2f_{n-2}\right]. \tag{1}$$
3. Is there any reason for distrusting this numerical scheme for solving $x' = f(t, x)$?
$$x_n - 3x_{n-1} + 2x_{n-2} = h\left[f_n + 2f_{n-1} + f_{n-2} - 2f_{n-3}\right]. \tag{2}$$
   Explain.
4. Convert the system of second-order ordinary differential equations
$$\begin{cases} x'' - x'y = 3y'x\log(t), \\ y'' - 2xy' = 5x'y\sin(t). \end{cases}$$
   Into a system of first-order equations in which t does not appear explicitly.
5. Write an autonomous system of first-order equations equivalent to
$$\begin{cases} x''' - \left[\sin(x'') + e^t x'\right]^2 + \cos(x) = 0, \\ x(0) = 3, \quad x'(0) = 4, \quad x''(0) = 5. \end{cases} \tag{3}$$
